Bose Einstein Condensation in the higher band of a time-dependent double well optical lattice

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Outline

- Introduction
  - Bose Einstein Condensate
  - Optical Lattice

- Motivation

- Theoretical Understanding
  - Wave functions and the Optical Lattice
  - Energy Bands
  - Formation of the Bose Einstein Condensate
  - Rethermalization
  - Decay to the first Band

- Summary

1925: Einstein's Prediction for the Ideal Bose Gas

In a system of non-interacting Bosons, a large fraction of the particles (N_0/N) condense to the ground state when cooled below a certain temperature (T_c).

$$T > T_c \qquad \frac{N_0}{N} \to 0$$

$$T < T_c \qquad \frac{N_0}{N} \to 1 - \left(\frac{T}{T_c}\right)^{3/2}$$

For $T < T_c$ the system has formed a Bose-Einstein condensate in the ground state $\vec{p} = 0$.

Major difficulty encountered in the formation of a Bose Einstein Condensate

If the atoms get too close, they may well form molecules!
In fact at the density and temperature predicted by Einstein,

$$\rho\lambda_{dB}^{3} = \zeta(3/2) = 2.612 \qquad \lambda_{dB} = h / \sqrt{2\pi m k_B T}$$

almost all materials are in the solid state.

Thus, the experiment had to start with a hot gas and cool it in such a way that the atoms simply didn’t condense into into liquids or solids.

This requires keeping the density of the atomic gas very low- about a million times less dense than the density of air at the Earth’s surface.

This in turn requires a very large wavelength, which in turn requires ultra-cold temperatures.

Experimental Evidence

In 1995, Bose Einstein Condensate was first produced in a vapor of Rb^{87} atoms that was confined by magnetic fields in the group of Eric Cornell and Carl Wieman at JILA.

This was closely followed by the group of Wolfgang Ketterle at MIT with Na^{23} atoms and the group of Randy Hulet at Rice Unversity with Li^{7} atoms.

Atomic Hydrogen has been condensed in 1998 at MIT in the group of Dan Kleppner.

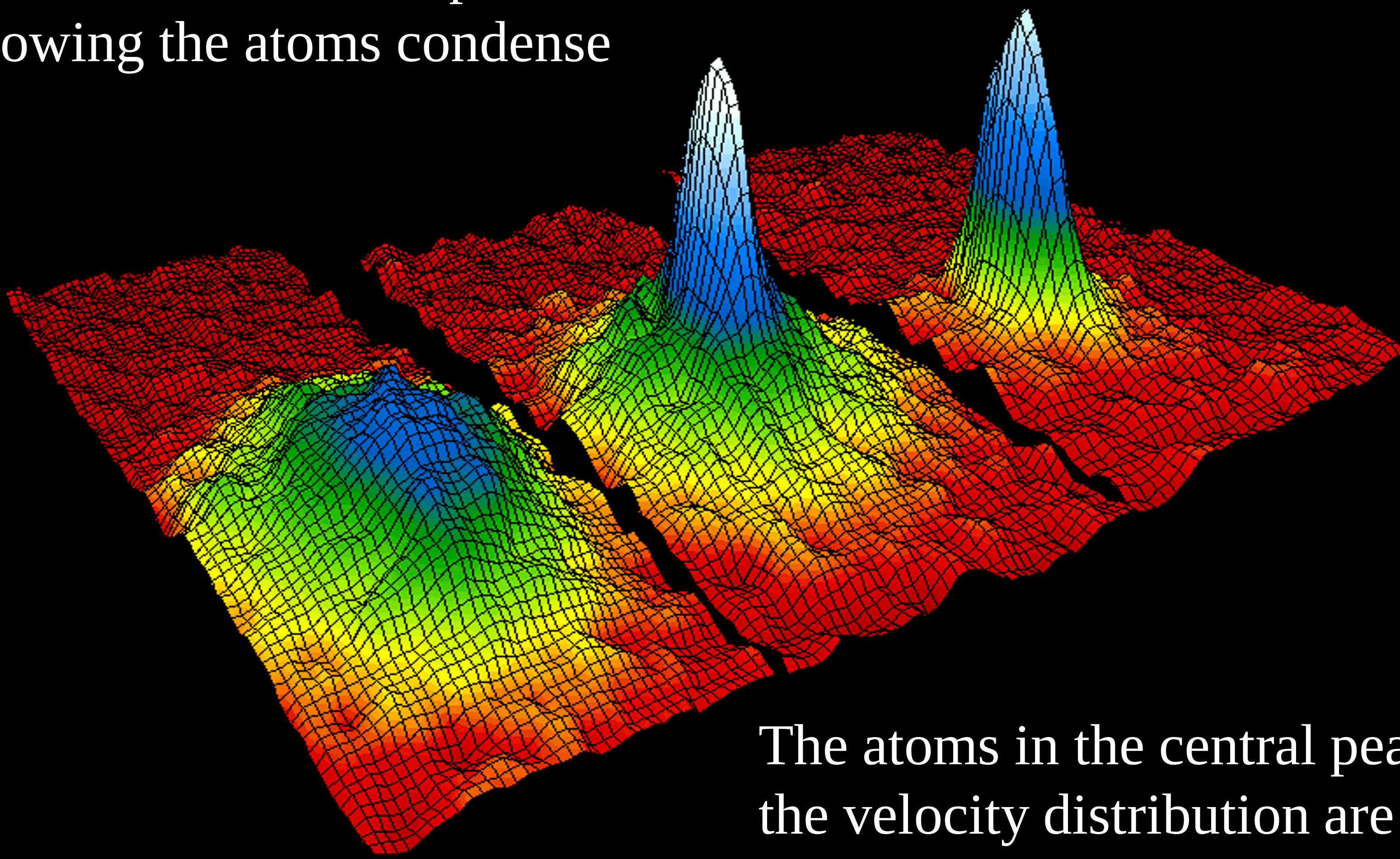
Typically one has in the experiments with condensates:

$$\rho < 10^{15}\, atoms / cm^3$$

$$T < 1 \mu K$$

Bose-Einstein Condensate

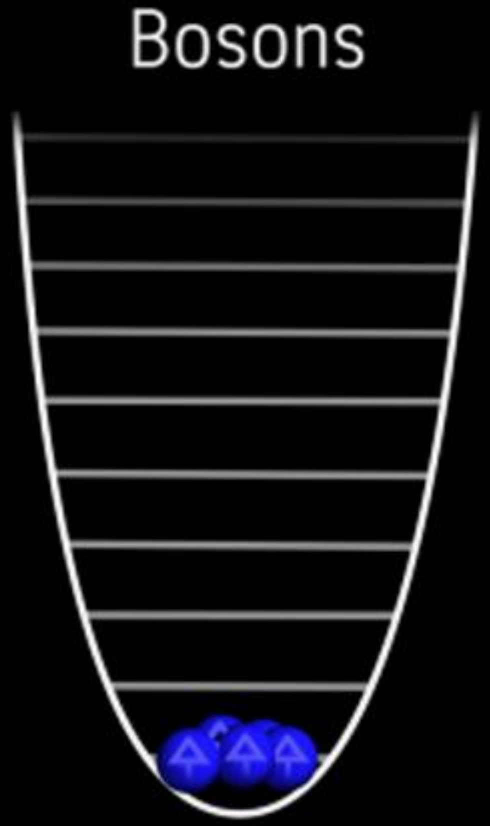
3-successive time snap shots
showing the atoms condense



The atoms in the central peak of the velocity distribution are in a single quantum state defined by the trap confining the atoms.

Ideal Bose Gas in a Trap

Let us consider the case of a harmonic trapping potential

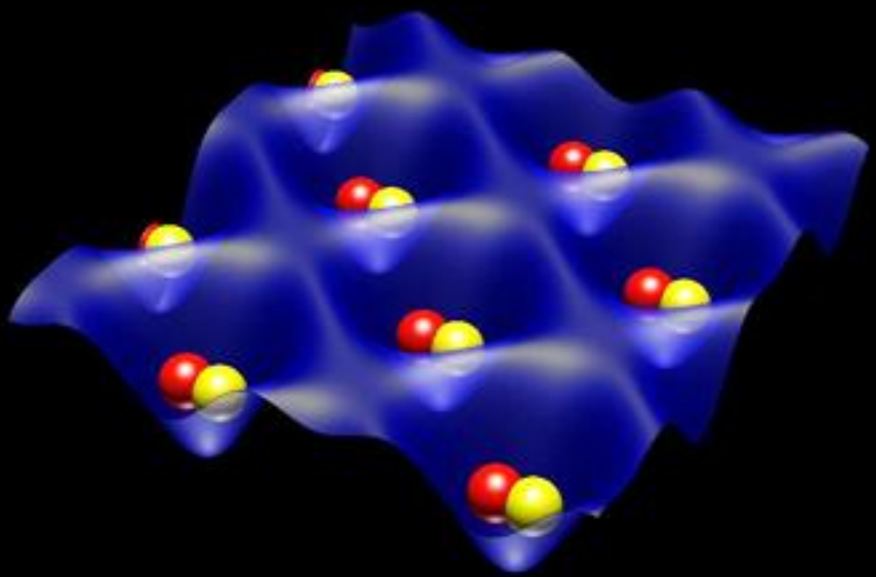


When a BEC forms, most of the atoms occupy the ground state of the well.

The fraction of condensed atoms is given by:

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c}\right)^3$$

Optical Lattice



Optical lattices are crystals made from a set of standing wave lasers.

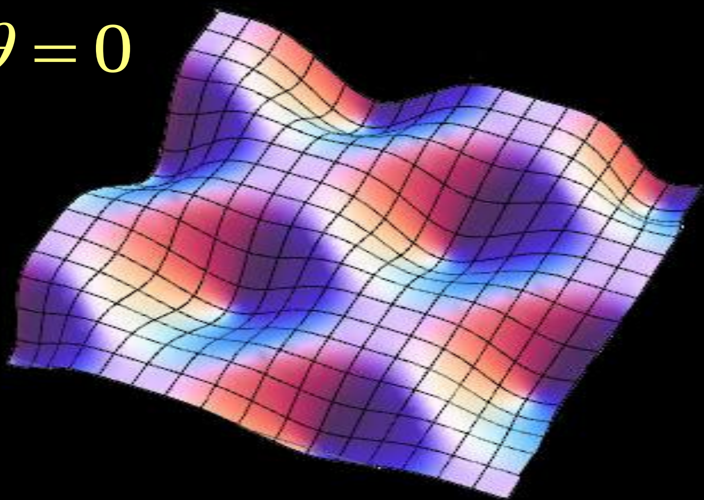
The electric fields of these lasers can interact with atoms-the atoms see a potential and therefore congregate in the potential minima.

There are two important parameters:

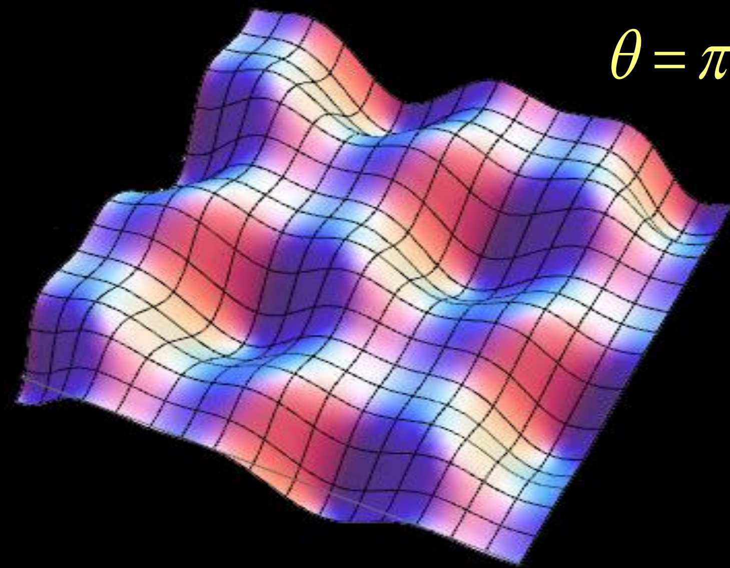
a. Well depth- depends on the power of the laser and phase difference between the standing waves.
b. Periodicity- depends on the wavelength of the laser

A Bipartite Square Optical Lattice

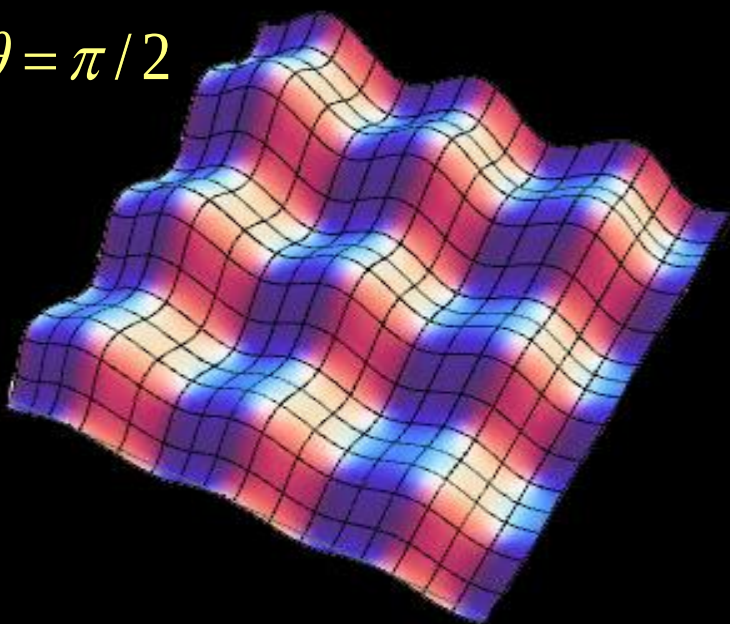
$\theta = 0$

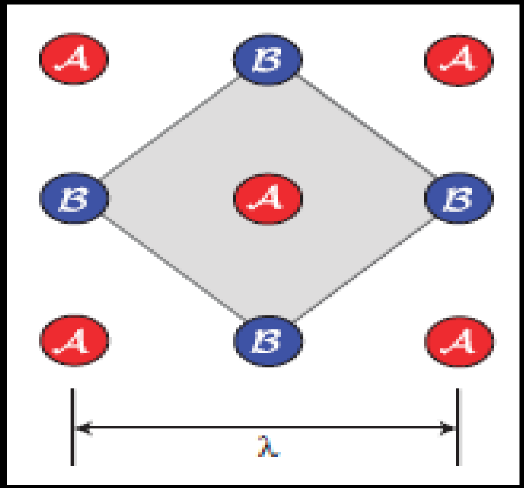


$\theta = \pi / 3$

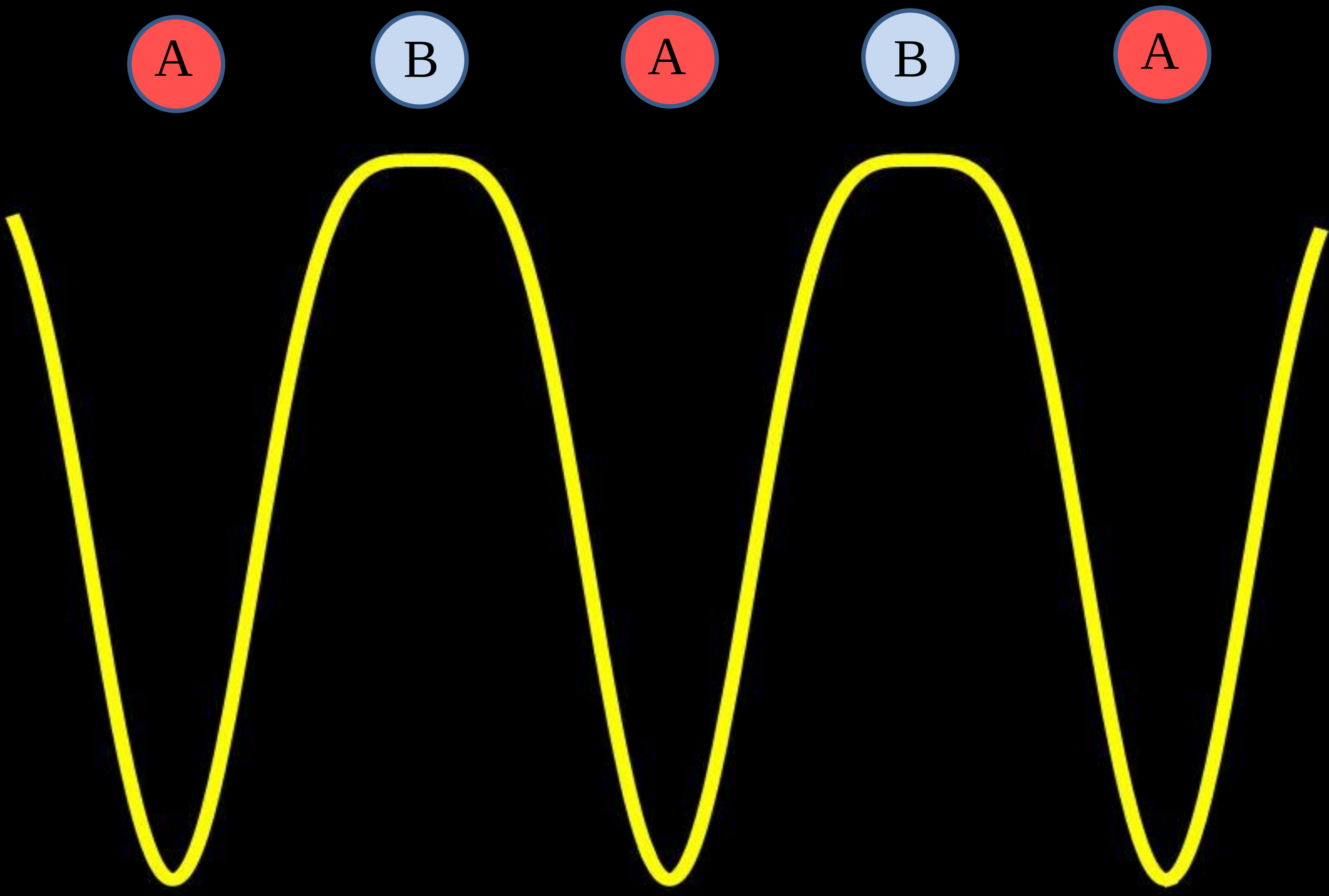


$\theta = \pi / 2$



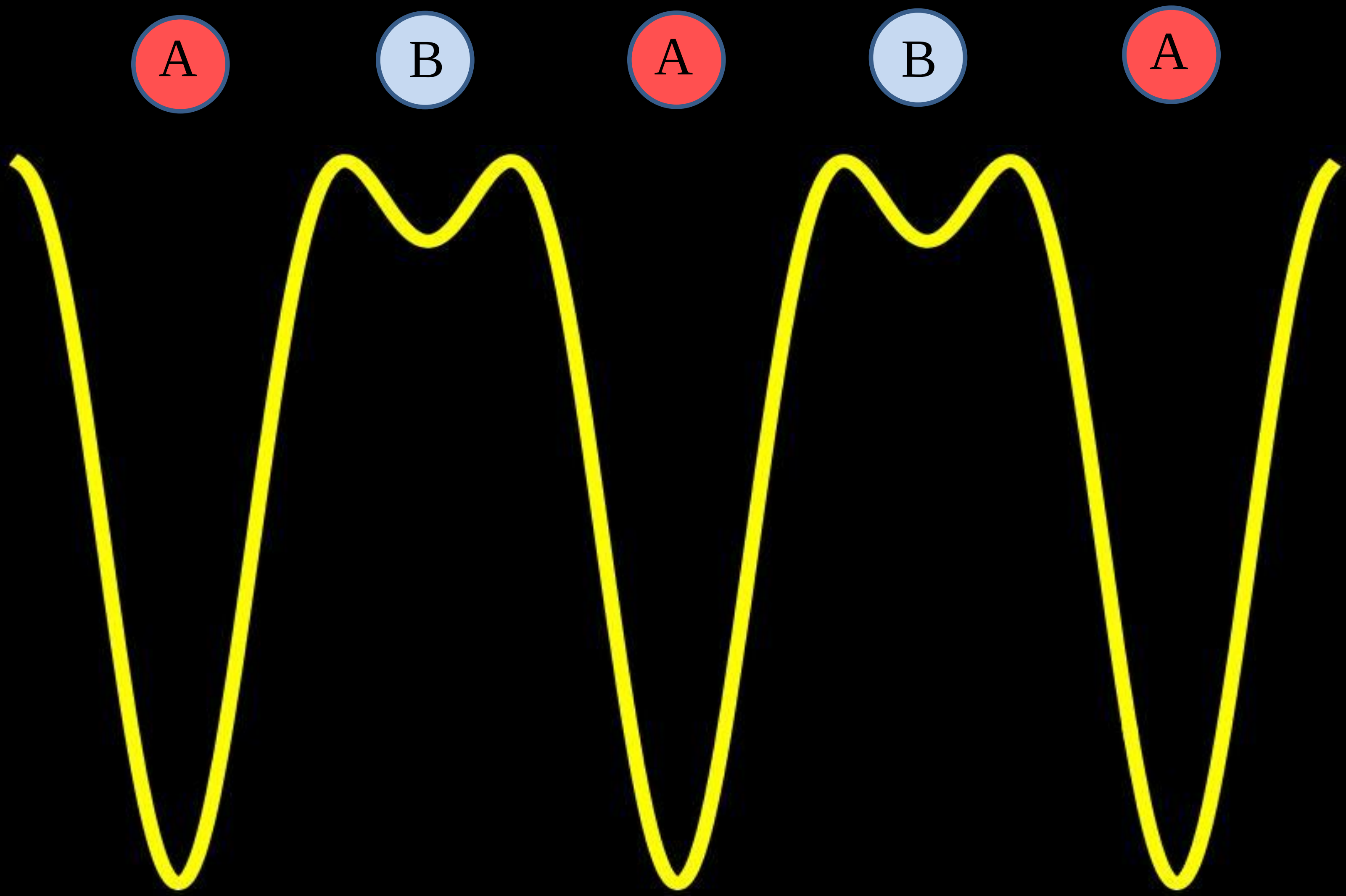


Changing the potential real time



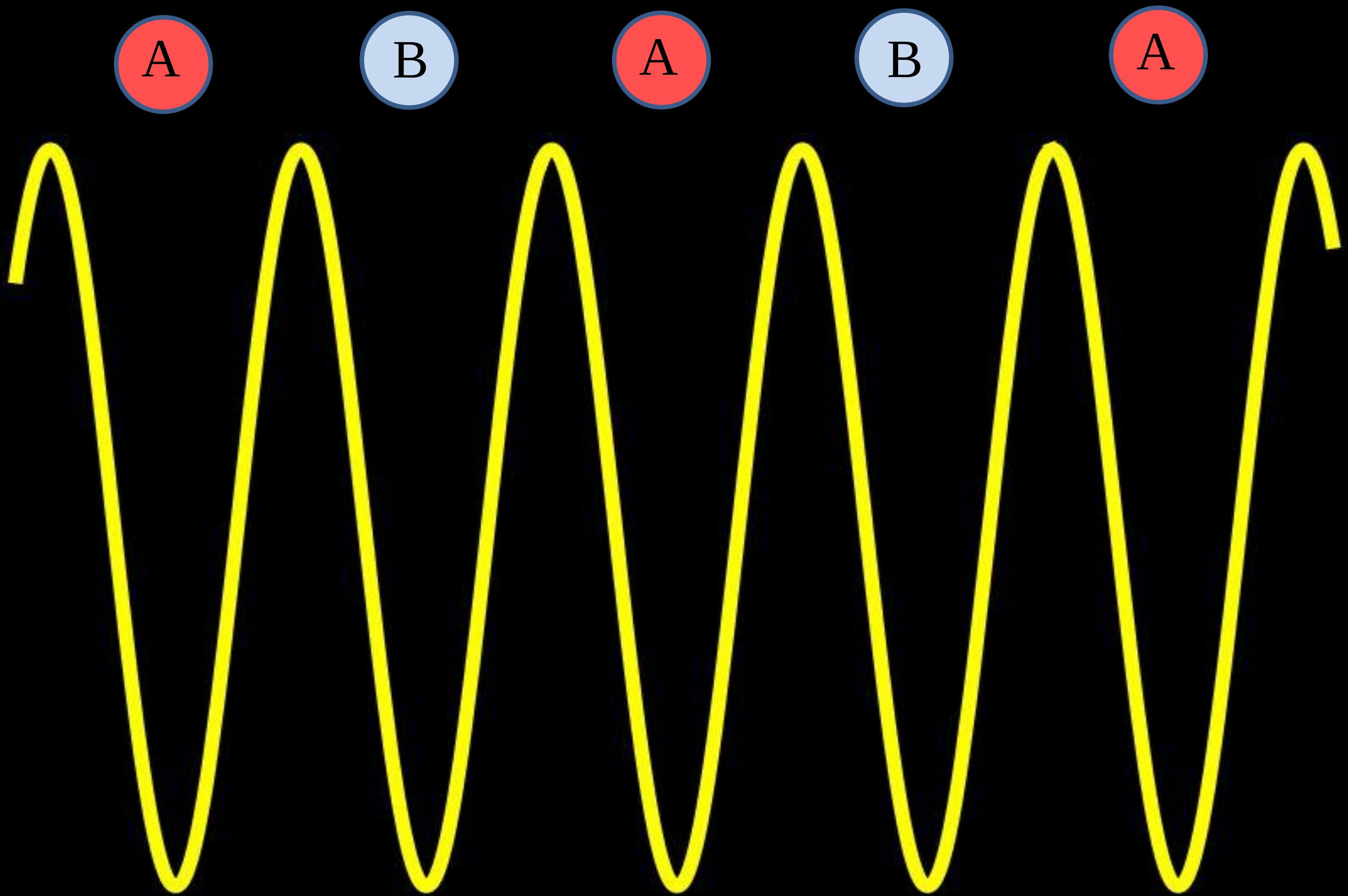
$\theta = 0$

Changing the potential real time



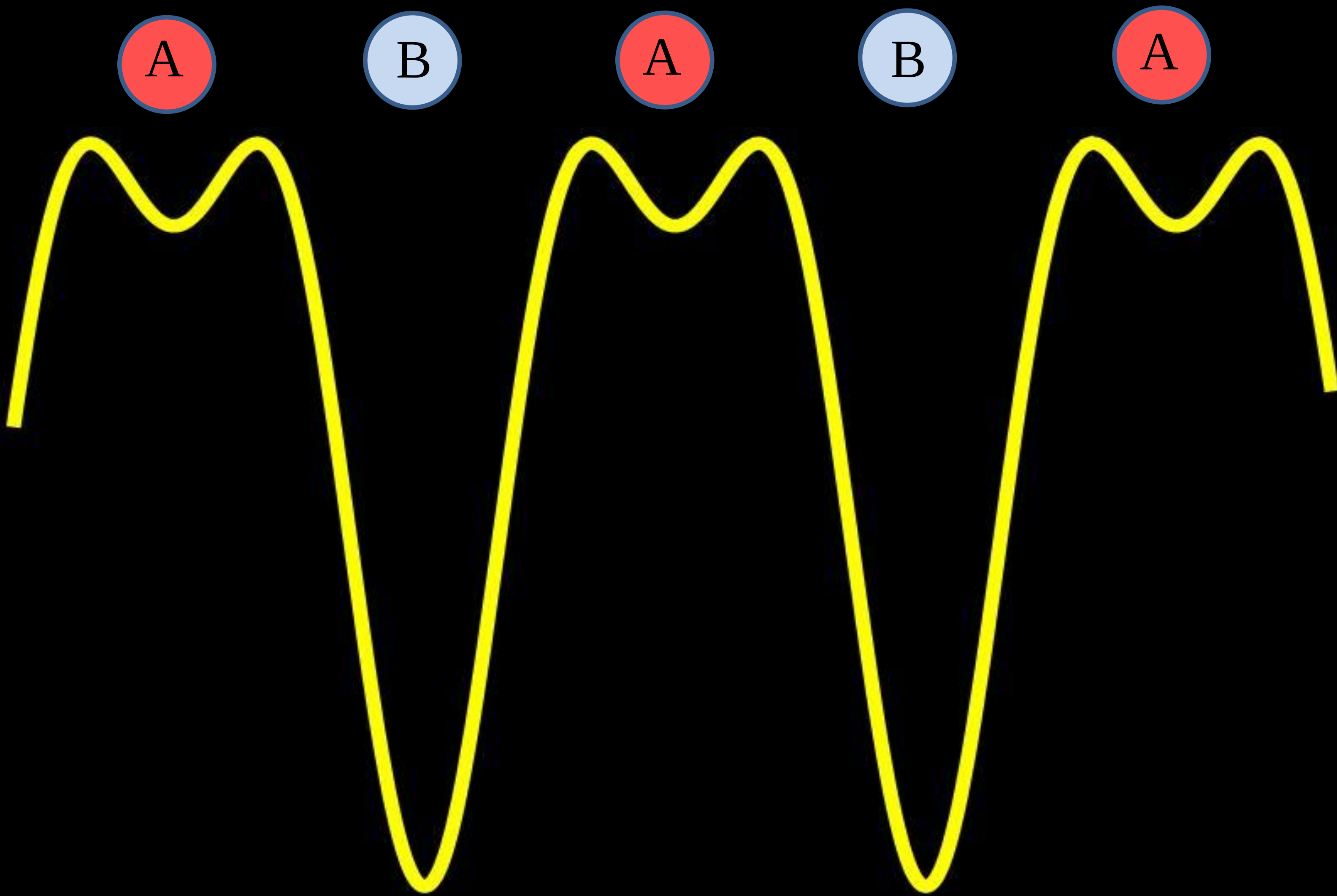
$$\theta = \pi / 3$$

Changing the potential real time



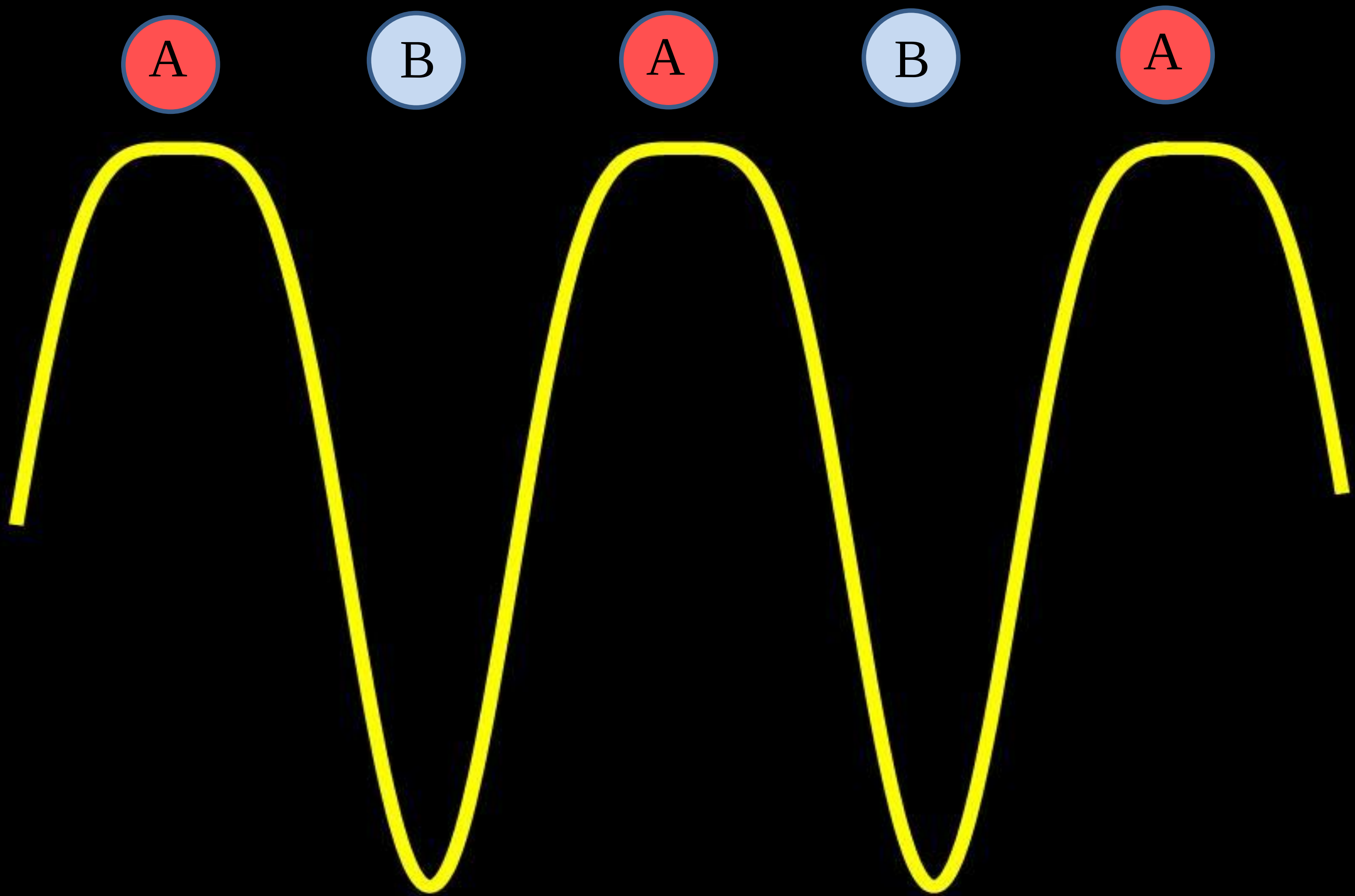
$\theta = \pi / 2$

Changing the potential real time



$\theta = 2\pi / 3$

Changing the potential real time



$\theta = \pi$

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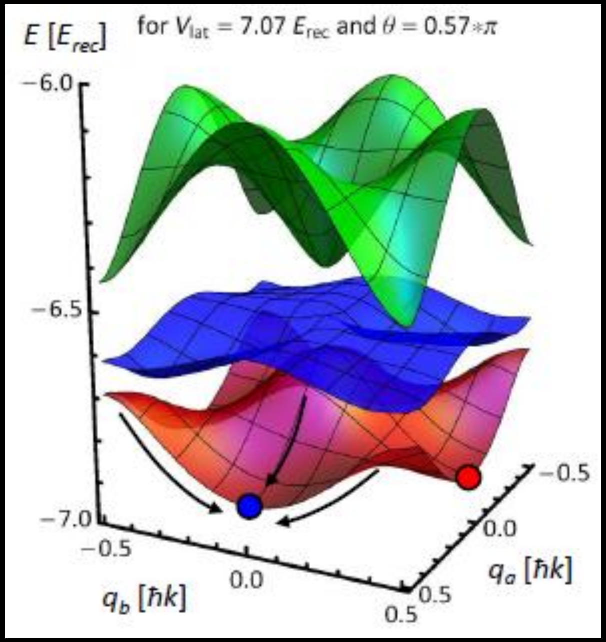
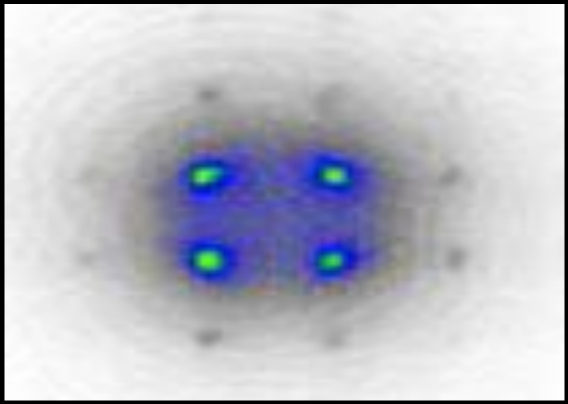
Condensation at finite momenta



$\theta = 0.57\pi$



a. Mix of local S-orbits at shallow A sites and local P-orbits at deeper B-sites.

b. Condensation at finite momenta on the edge between the first and second BZ.

c. Decay to first band with lifetime of 110 ms.

d. Absence of zero momentum peak.

G. Wirth et al., Nature Physics doi:10.1038/nphys1857

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The Trapping Potential

Let us write down the mathematical form for the trapping potential that we have already discussed at length:

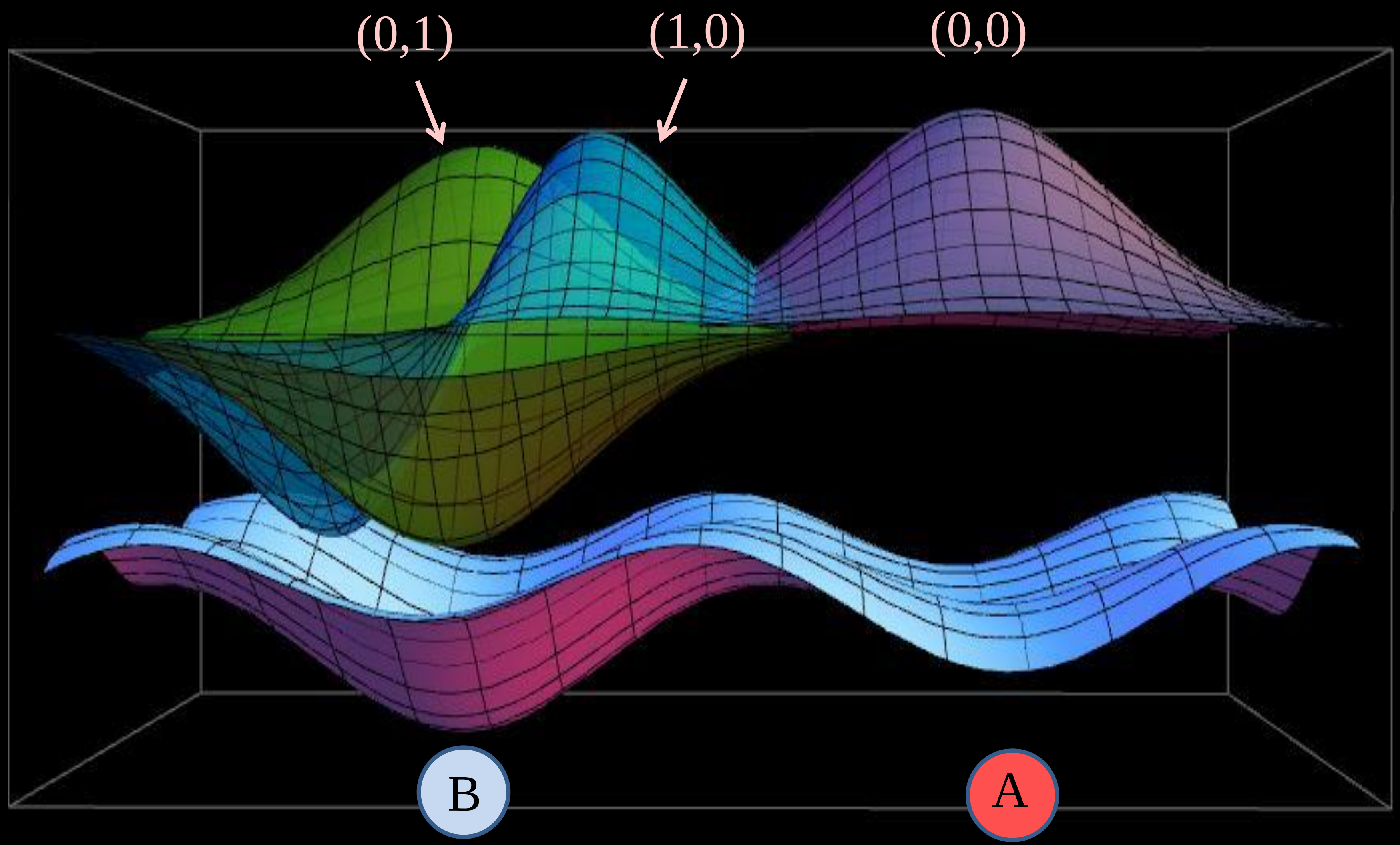
$$V(x,y) = -|V_0|\left(\cos^2(kx) + \cos^2(ky) + 2\cos\theta\cos(kx)\cos(ky)\right)$$

We can Taylor expand this potential around the shallower and deeper wells, and to second order, it will look exactly like the simple harmonic oscillator potential. Thus, we can choose the Wannier functions to be simple harmonic oscillator solutions.

In addition, it is important to note that there is also a weak harmonic trapping potential in the z-direction.

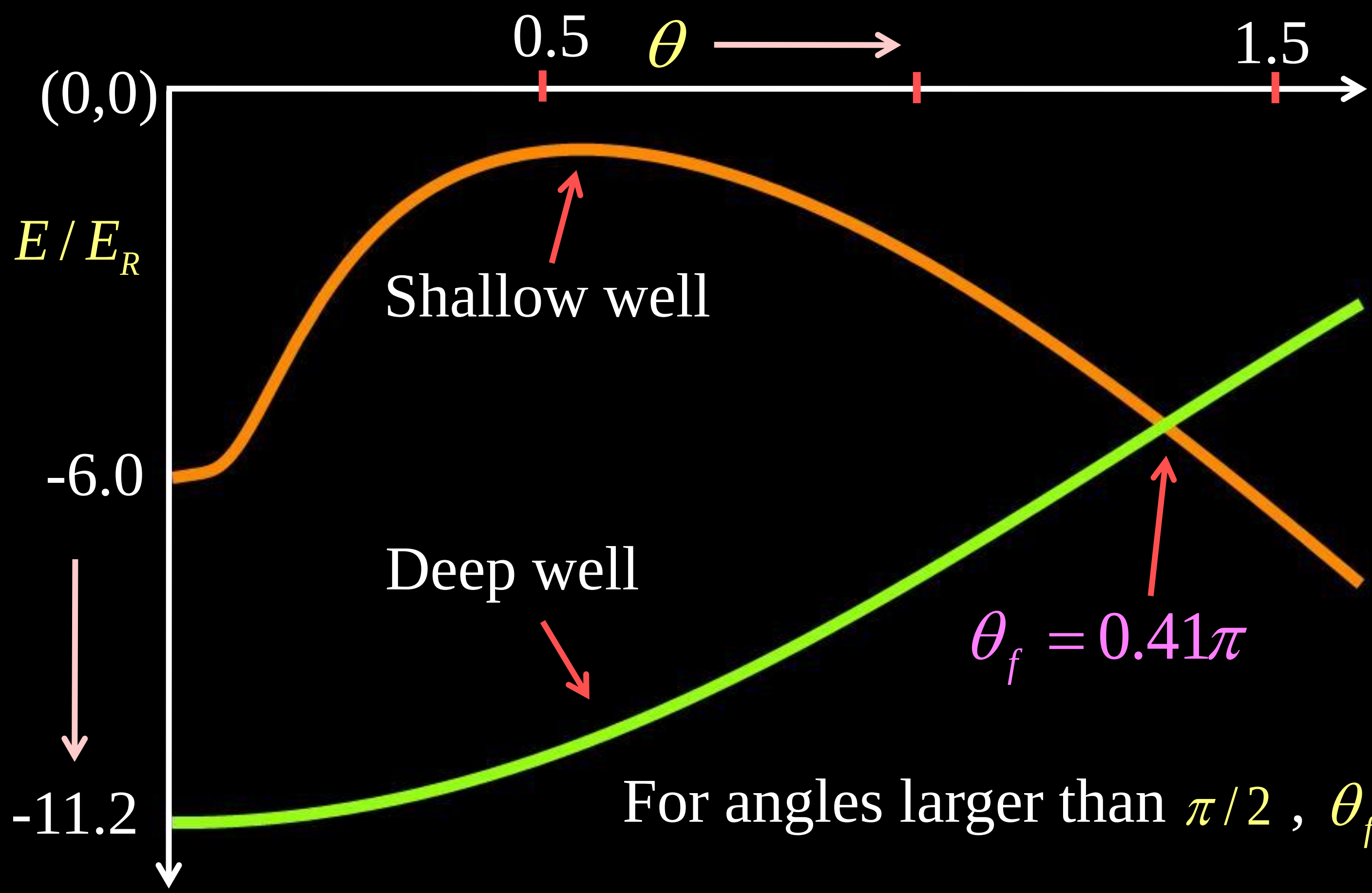
At each lattice site, the confinement is much stronger in the x & y direction than the z-direction, a fact that we will use later on.

Wave functions at the A and B lattice sites



Energy at each site

a. The energy at lattice each site depends on the value of "θ"
b. The experiment is performed at a value of "θ" where the lowest energy S-state in the shallow A wells are tuned into resonance with the P-state of the deeper B wells.



For angles larger than $\pi / 2$, $\theta_f = 0.59\pi$

Tight Binding Approximation

$$H = \begin{pmatrix} E_0 & -2it\sin(k_x a) & -2it\sin(k_y a) \\ 2it\sin(k_x a) & E_1 & 0 \\ 2it\sin(k_y a) & 0 & E_1 \end{pmatrix}$$

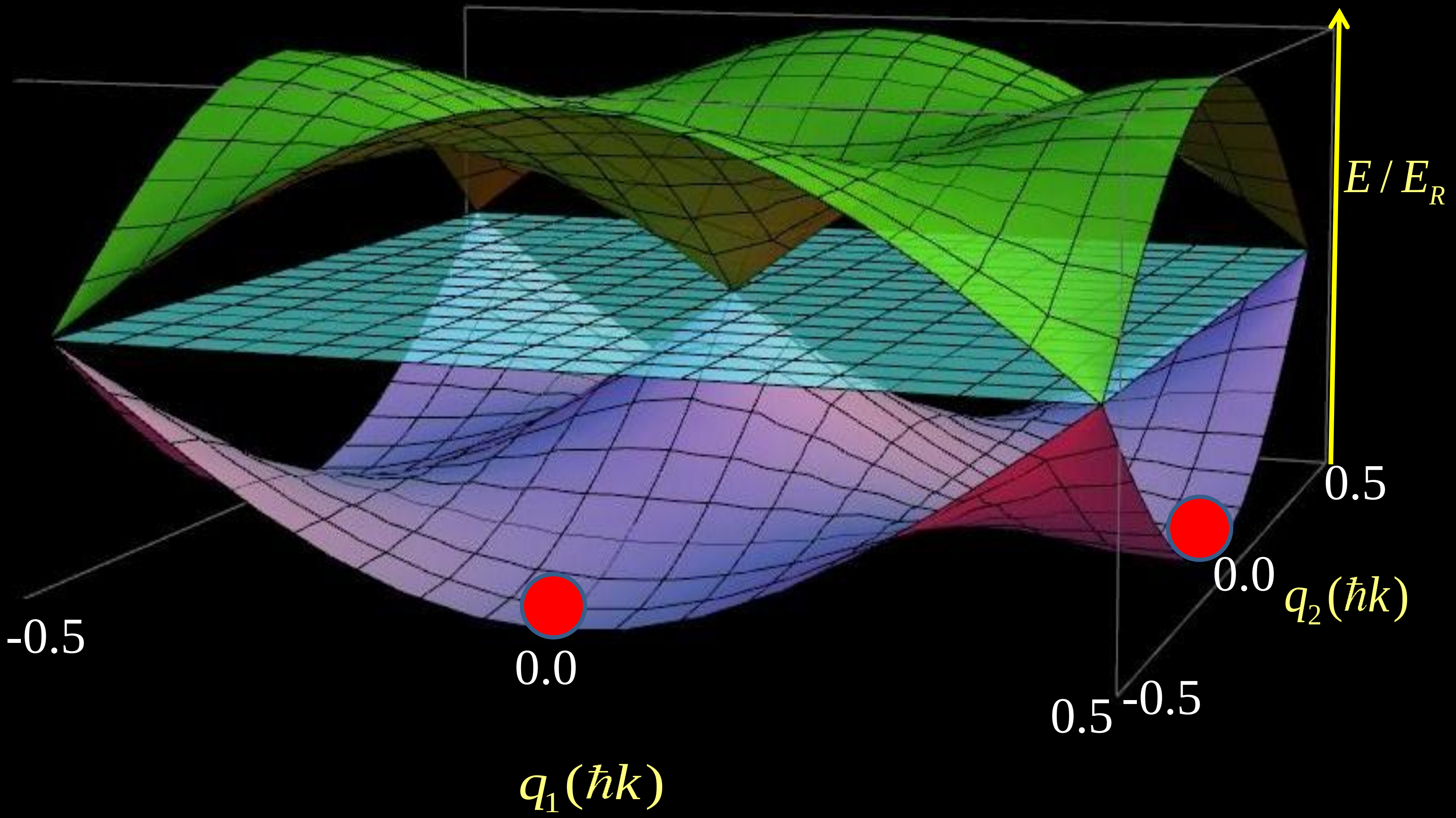
$E_0 \longrightarrow$ On-site energy in the shallow well

$E_1 \longrightarrow$ On-site energy in the neighboring deeper well

$t \longrightarrow$ Hopping term $= \left\langle \psi_i \middle| \hat{H} \middle| \psi_j \right\rangle$

$E_0 \approx E_1$

Energy Band Diagram



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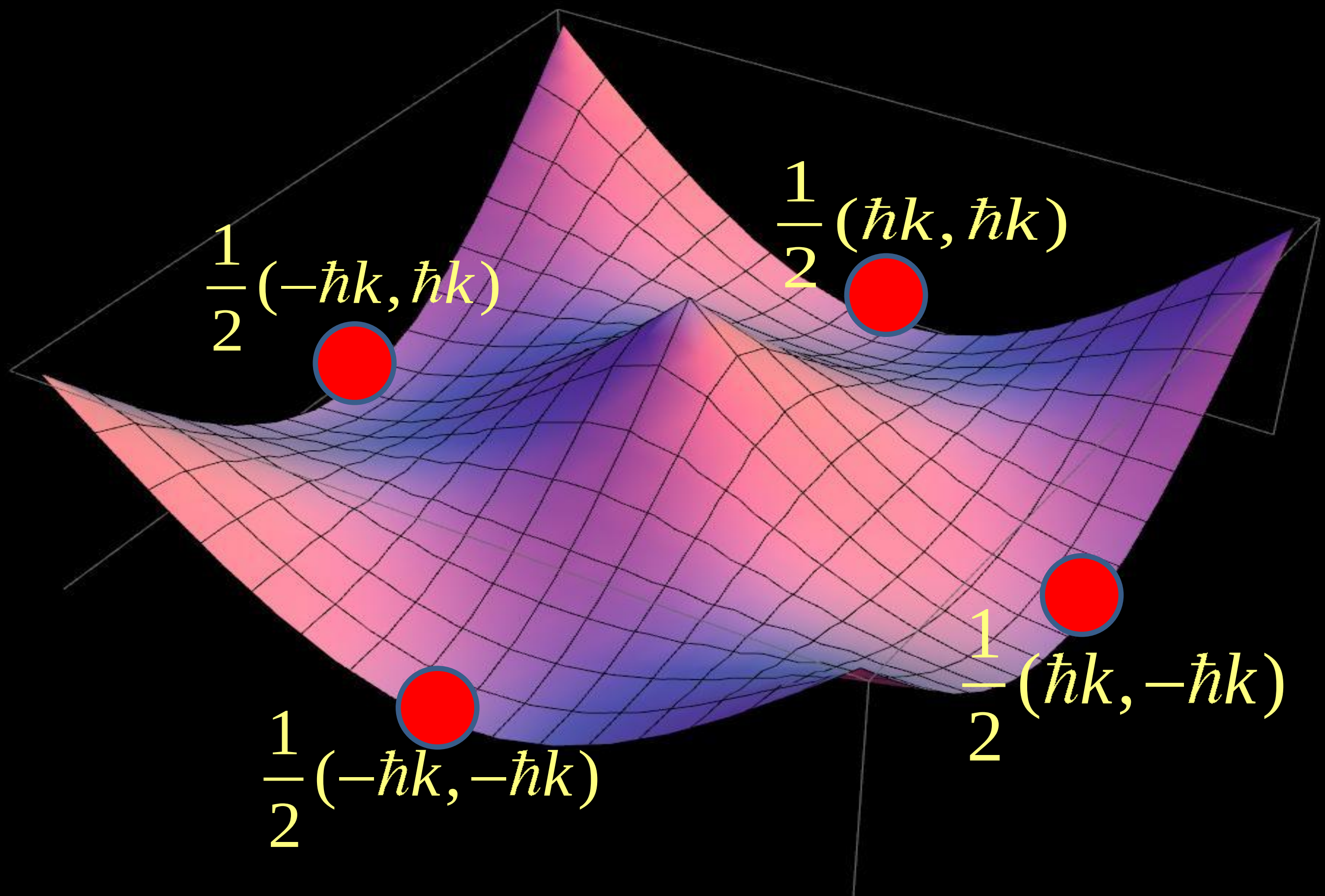
- Conclusion

Studying the formation of the Bose Einstein Condensate



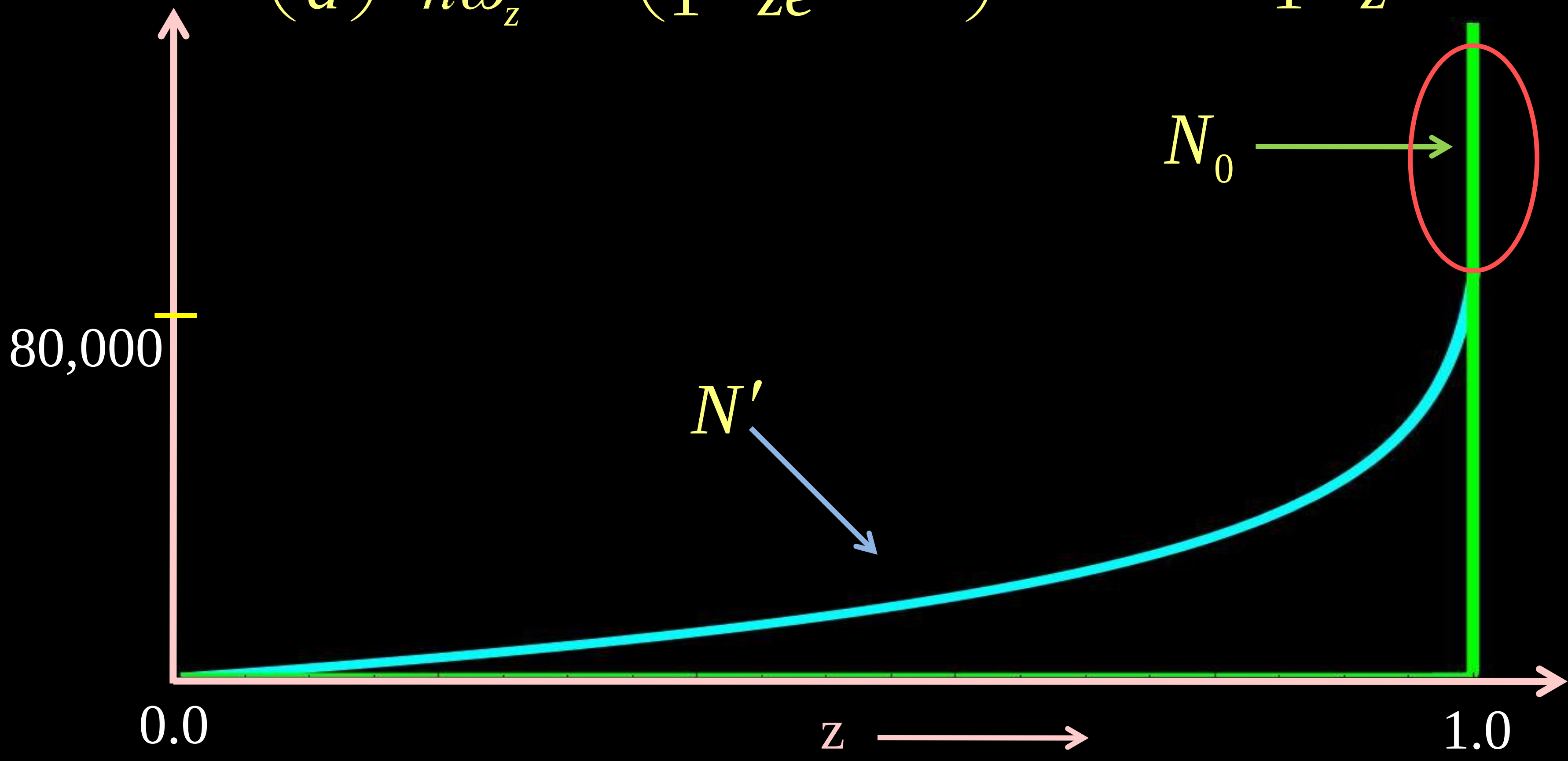
$$n_\varepsilon = \frac{1}{(1/z)e^{\beta\varepsilon} - 1}$$

$$N_o = \frac{z}{1 - z}$$

Formation of the Bose Einstein Condensate

The resulting math will lead us to the following two quantities:

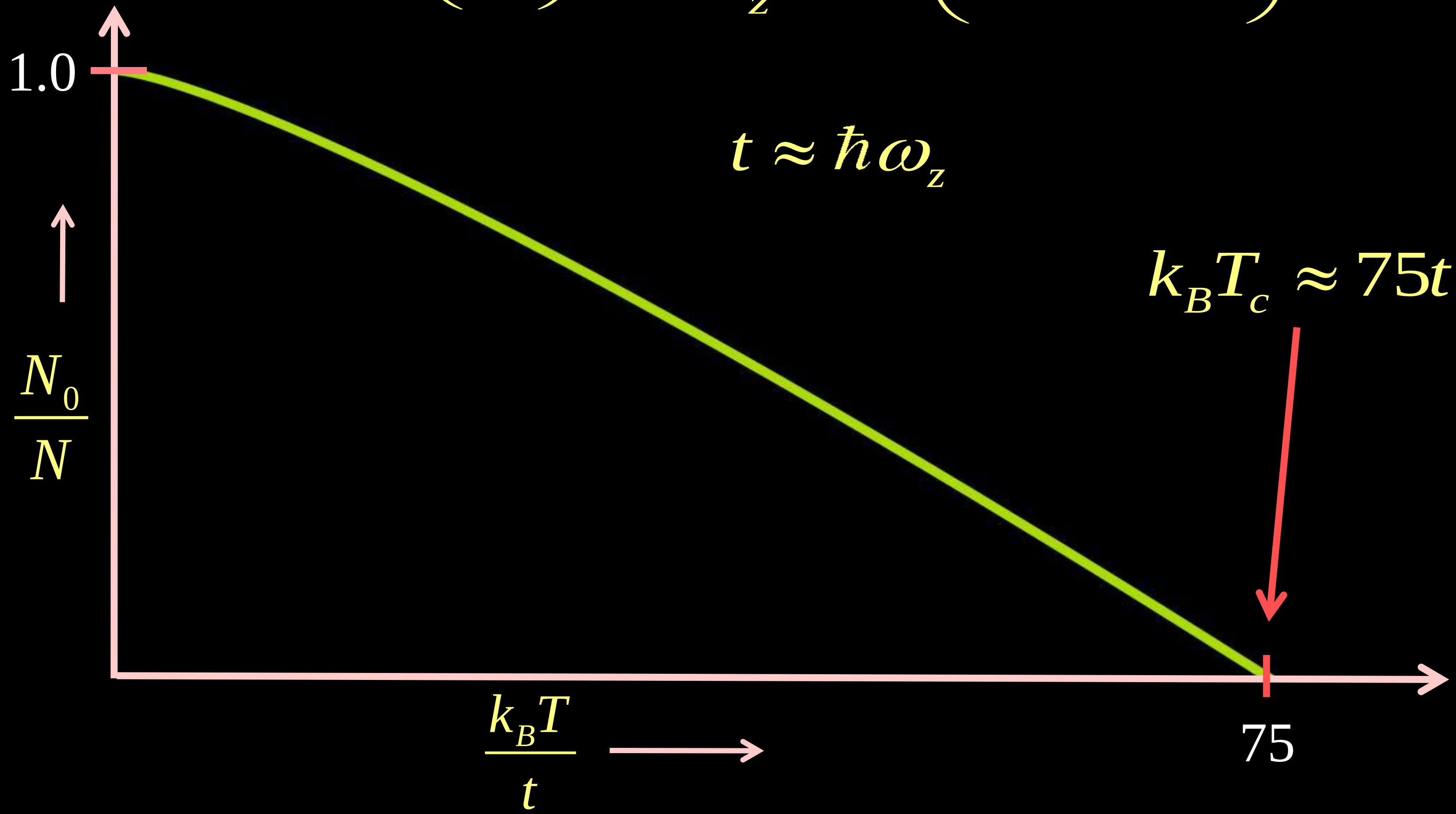
$$N' = \left(\frac{l}{a}\right)^2 \frac{k_B T}{\hbar \omega_z} \log\left(\frac{1}{1 - z e^{-\beta t/\sqrt{2}}}\right), \qquad N_0 = \frac{z}{1-z}$$



Condensed fraction and critical temperature

The fraction of population in the condensate is given by:

$$\frac{N_0}{N} = 1 - \left(\frac{l}{a}\right)^2 \frac{k_B T}{\hbar\omega_z} \log\left(\frac{\sqrt{2} k_B T}{t}\right) \frac{1}{N}$$



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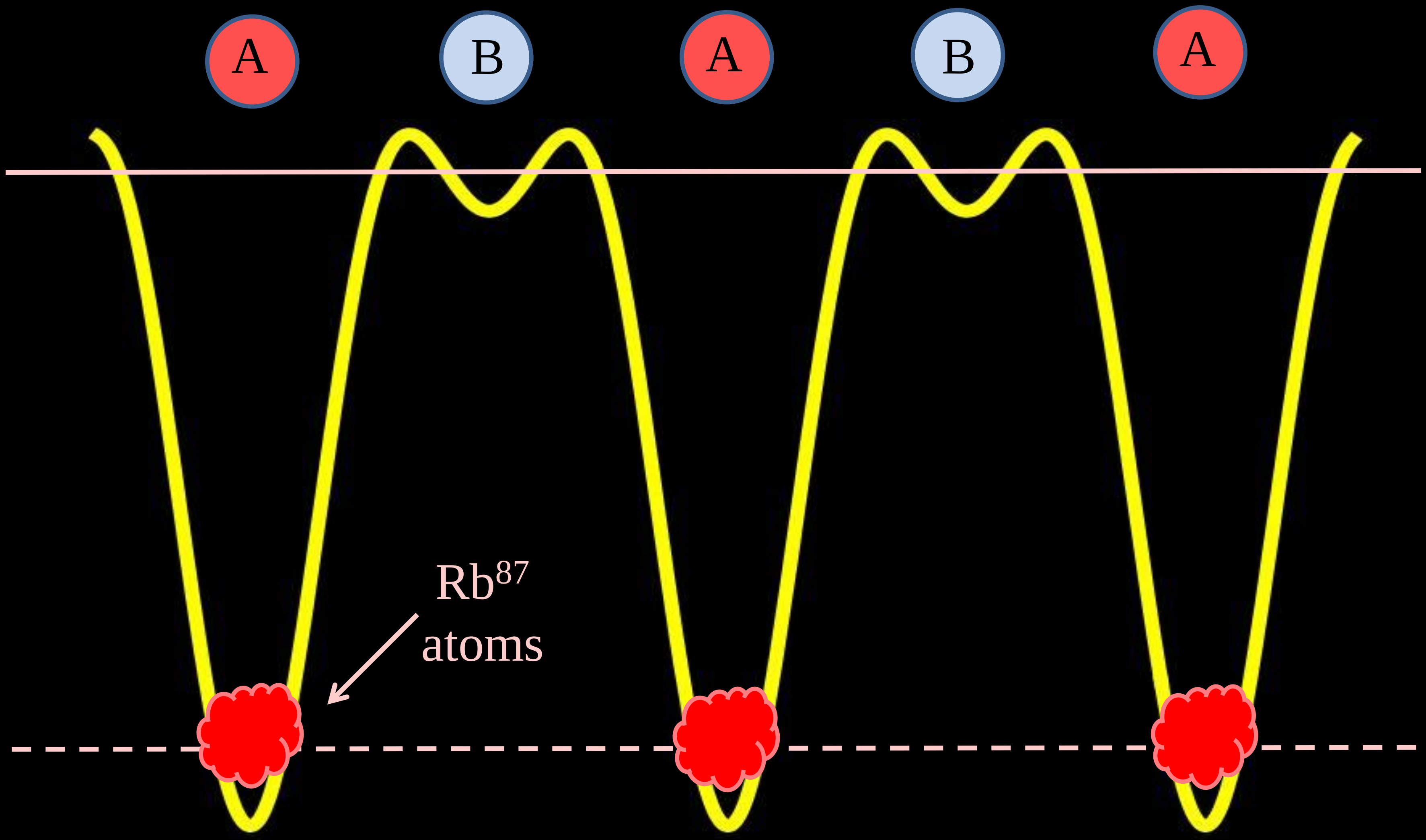
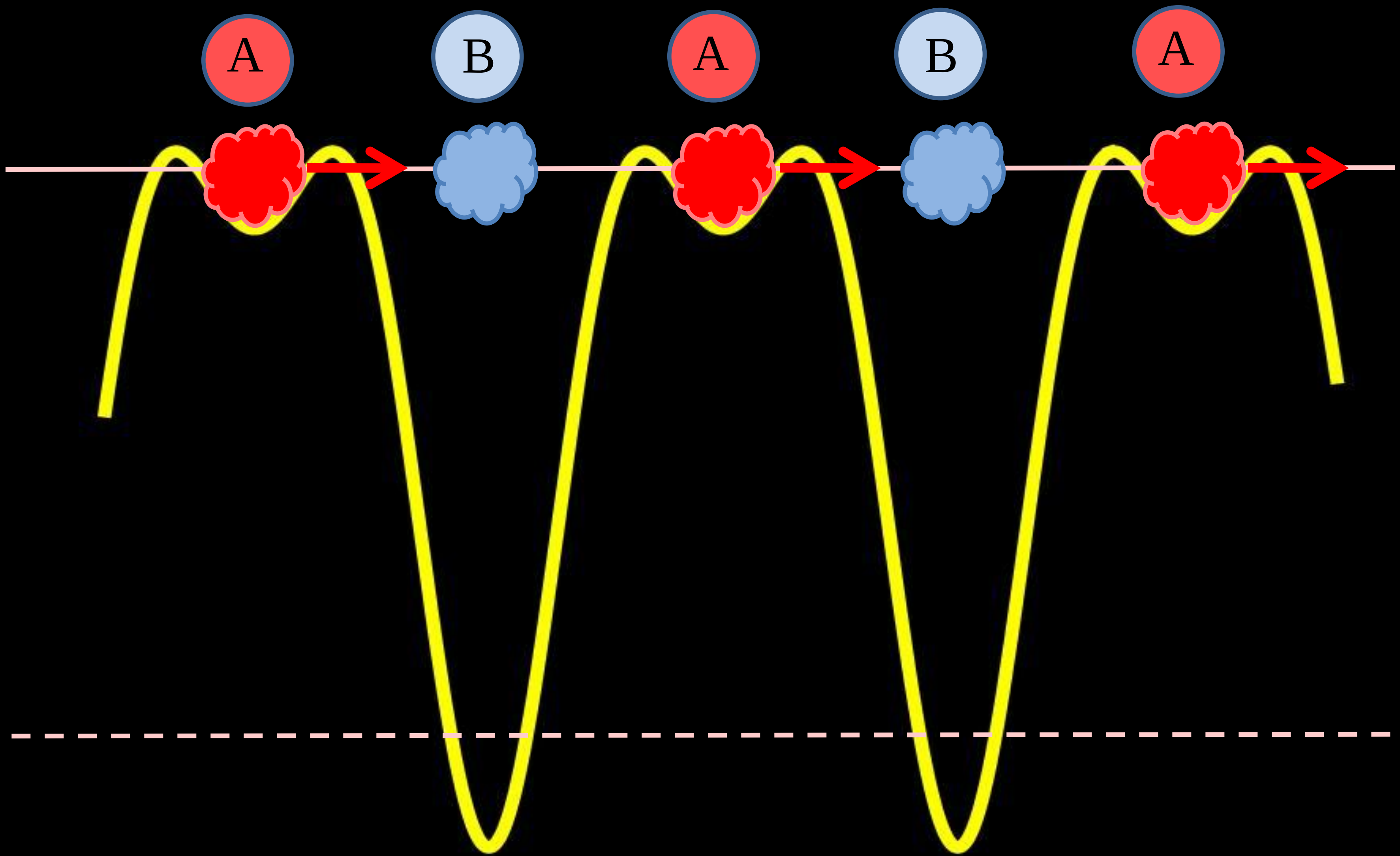
How do they populate the excited bands? Step#1



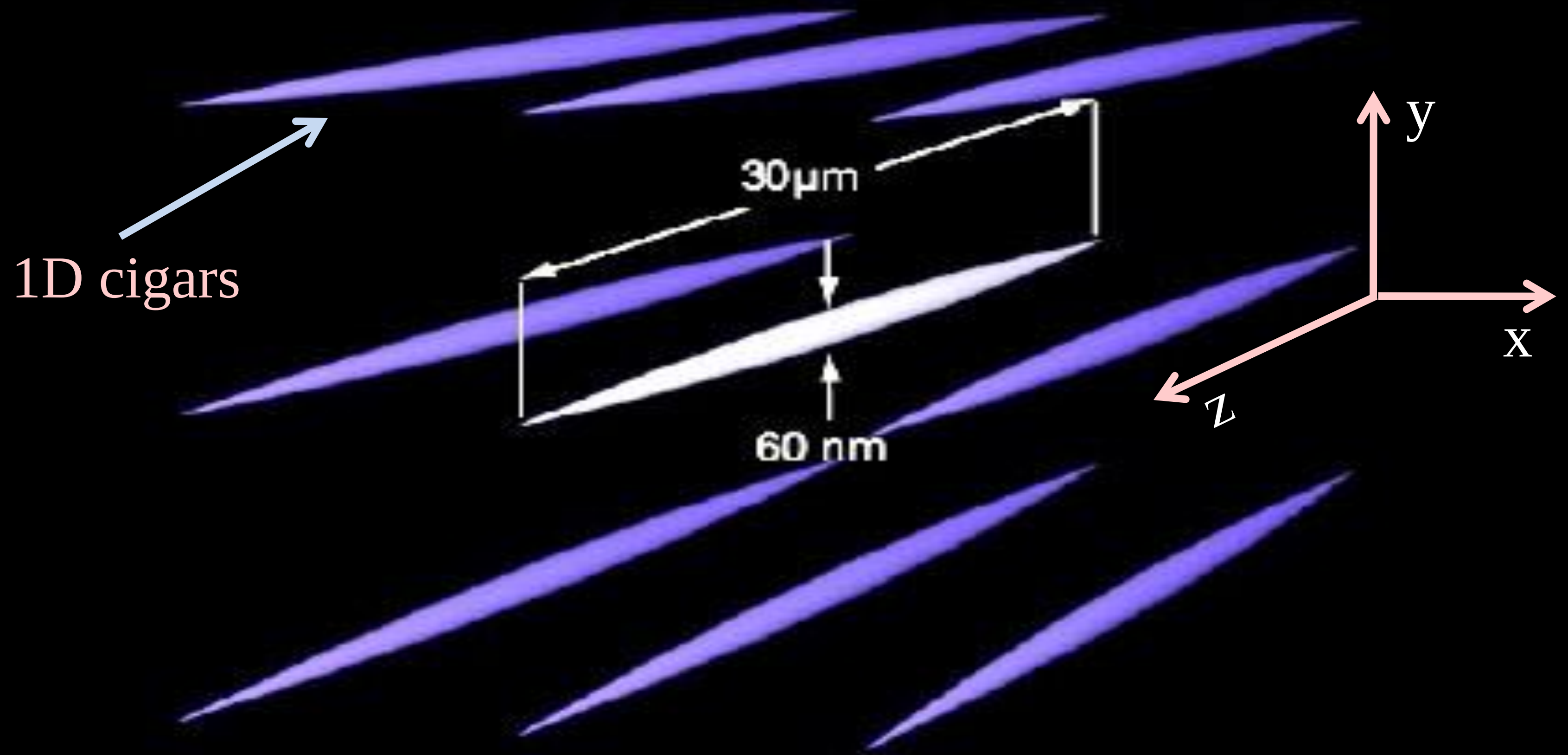
$\theta < \pi / 2$

How do they populate the excited bands? Step#2



$\theta > \pi / 2$

Transition from 1D to 3D



a. Before the onset of tunneling, the atoms are confined to these "1D cigars", and they do not talk to each other.
b. After the onset of tunneling, the atoms could hop around between these cigars, and thus we make a transition from a 1D to a 3D system.

Rethermalization

Thus, we start with a bunch of 1D systems in thermal equilibrium, say with an initial temperature T_i.

With the onset of tunneling, the atoms interact among themselves, and rethermalize, say to a final temperature T_f.

How different is T_f from T_i ?

It turns out that if we do the math, we get the following:

$$T_f \approx \frac{T_i}{\sqrt{3}}$$

Thus, if the initial temperature T_i is already below T_c, the above condition should further facilitate the formation of a BEC in the final state.

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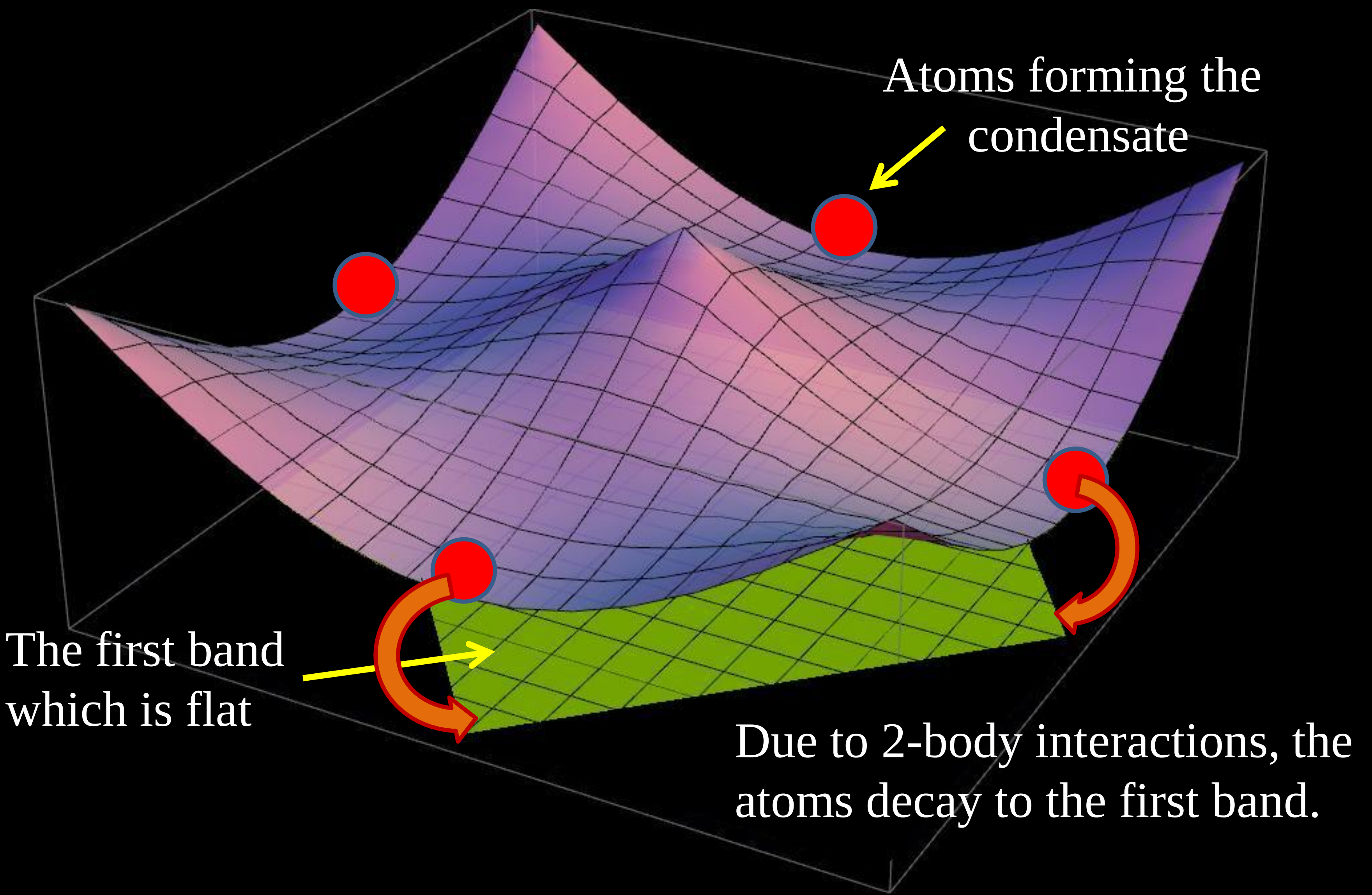
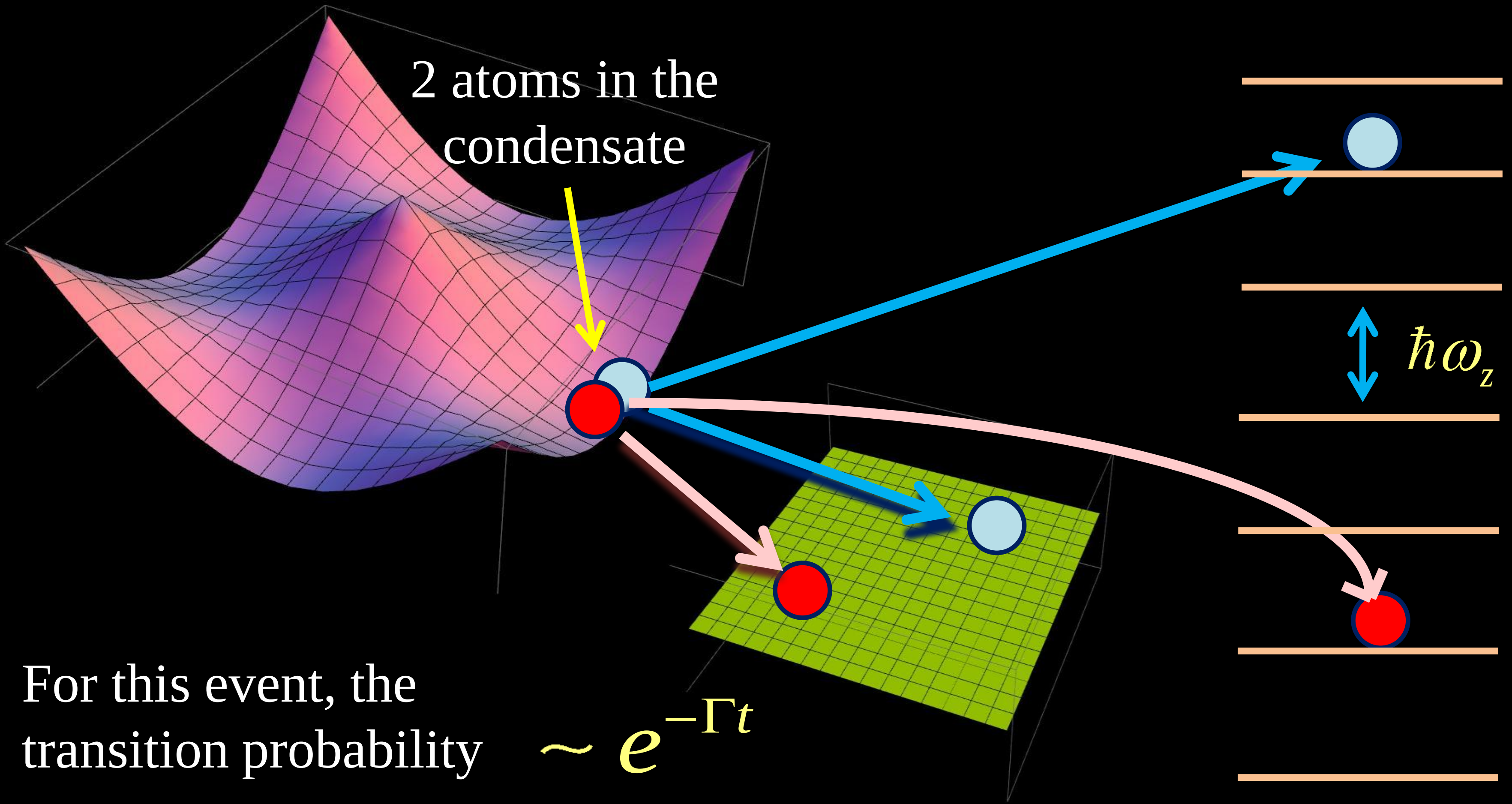
Decay to first band



Two body interactions

The dominant term



For this event, the transition probability $\sim e^{-\Gamma t}$

Therefore, the atoms decay into the first band with a time constant $1/\Gamma$

Summary

- We have discussed the formation of Bose Einstein condensate and its first experimental realization.
- There is recent experimental evidence of a condensate being formed in an excited band.
- The lattice used has shallower and deeper wells arranged in a chequerboard pattern, and the well depths can be varied real time.
- We have theoretically analyzed the formation of a BEC at four condensation momenta, at the edge of the first and second BZ.
- Our results show that rethermalization should further facilitate the formation of a condensate.
- The atoms in the condensate ultimately decay to the first band, and our results show that there is one dominant channel contributing to this decay.

THANKS