Quantum phases in an asymmetric double-well optical lattice

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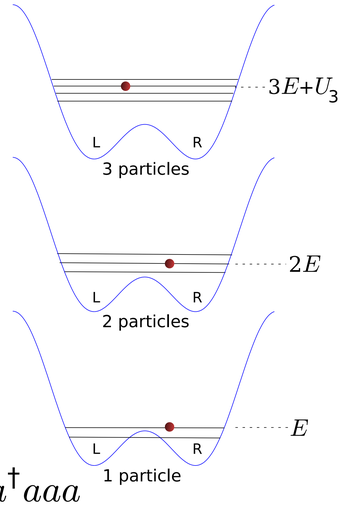
Motivation

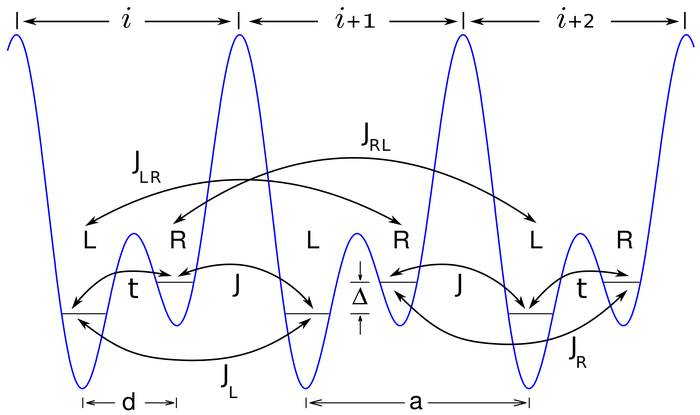
- Simulations of many-body systems with atoms in optical lattices.
- We are interested in lattice geometries with asymmetric double wells along one axis.
- This gives more general and richer Hamiltonian.
- Importantly, Hamiltonian includes tunneling energies beyond nearest neighbors.
- Also, interaction between atoms at adjacent wells become important.
- By tuning lattice parameters, the goal is to be able to select 2 & 3 particle states, such that there is no two-body interaction energy, but finite three-body interaction energy.



$$H_{\text{eff}} = Ea^\dagger a + \frac{U_3}{6} a^\dagger a^\dagger a^\dagger aaa$$

Double-well lattice

- Tunneling energies, $t \gg J > J_L \simeq J_R > J_{LR} > J_{RL}$.
- Very small lattice tilt, $t \gg \Delta$.
- Tunneling beyond nearest neighbor important.

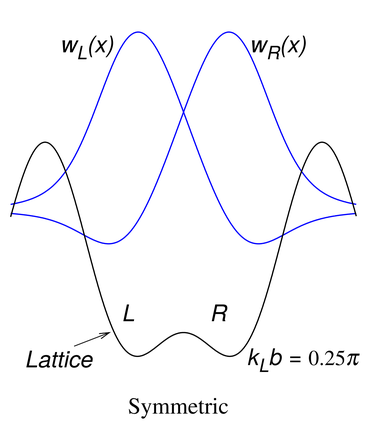


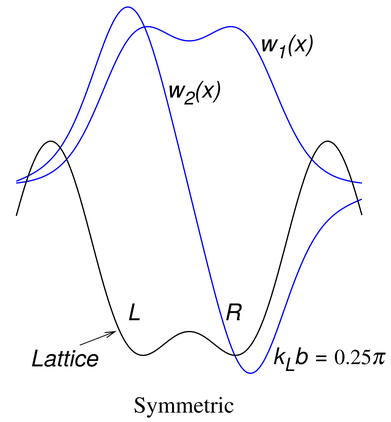
Wannier Functions

- Wannier functions $w(x)$ are obtained numerically.
- They are eigenfunctions of the position operator in basis of Bloch functions [1],

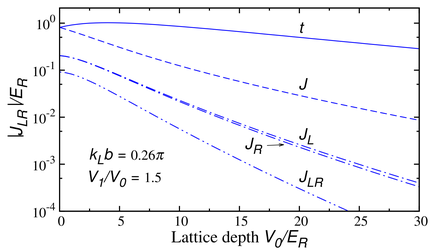
$$X_{k\alpha,k'\alpha'} = \int \psi^*_{k,\alpha}(x) x \psi_{k',\alpha'}(x)\, dx.$$

- Band index $1 \le \alpha, \alpha' \le 2$, i.e., first two bands.
- For $w_1(x)$ & $w_2(x)$, we set $\alpha = \alpha'$.
- For $w_L(x)$ & $w_R(x)$, we mix first two bands.



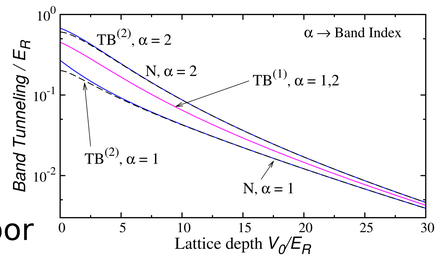


Tunneling Energies



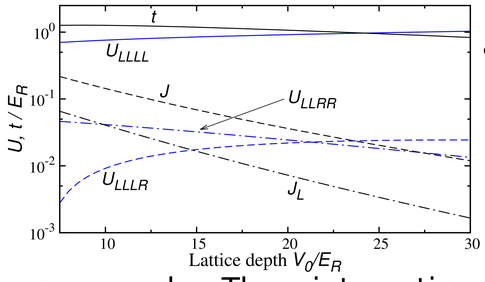
- Tunneling energies numerically obtained from the Wannier fns.
- $t = \langle w_L | \hat{H} | w_R \rangle$, etc.
- Plots are for slightly tilted lattice.

- Two approximations for *TB* Hamiltonian:
- $TB^{(1)}$, involving only nearest neighbor tunneling, t & J.
- $TB^{(2)}$, involving all relevant hoppings.
- $TB^{(1)}$ inadequate! Next-nearest neighbor tunneling a must.



Interaction Terms

- Bose-Hubbard Hamiltonian with two body interactions.



- For instance,

$$U_{LLLL} = g \int d\vec{r} w_L(\vec{r})^4.$$

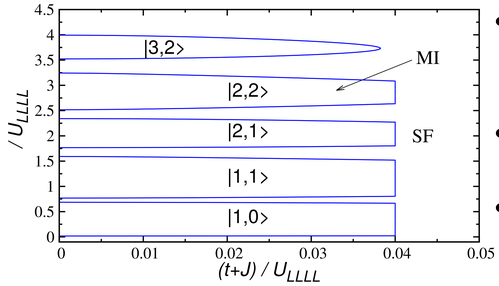
- U_{LLLR}, U_{LLRR}, J & J_L can be of same order. Thus, interactions between atoms at adjacent *L* & *R* wells are important.

SF to Mott Transition

- Bose-Hubbard Hamiltonian including all relevant tunneling and interaction terms.
- Use Decoupling approximation

$$a^\dagger_{i,L} a_{j,R} = \psi_L a_{j,R} + \psi_R a^\dagger_{i,L} - \psi_L \psi_R,$$

- $a^\dagger_{i,L}$, $a_{j,R} \rightarrow$ creation, annihilation operators,
- $\psi_L = \langle a_L \rangle$, $\psi_R = \langle a_R \rangle$, are order parameters.
- SF state is signified by non-zero order parameter.



- Effective hop along x, $t_{eff} = t + J$
- SF state for $t_{eff} \ge 4J_\perp$
- Chopped Mott lobes.

Outlook

- Determine 2 & 3 particle states for which there is no two body interaction but finite 3 body interaction.
- Find out suitable lattice transformations that would populate such selected states.

References

[1] Uehlinger *et al.*, PRL 111, 185307 (2013).